Implementation and use of .NET and Silverlight class library for numerical solution of optimal control problems

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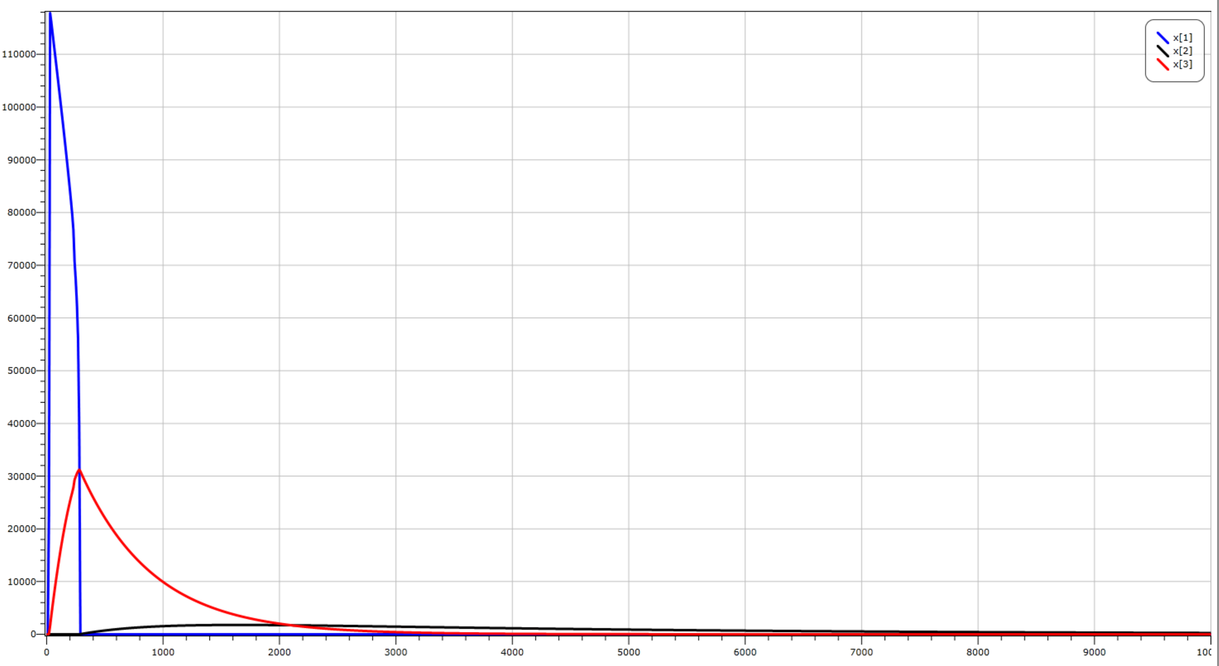
Computational base: Microsoft Research Solvers library

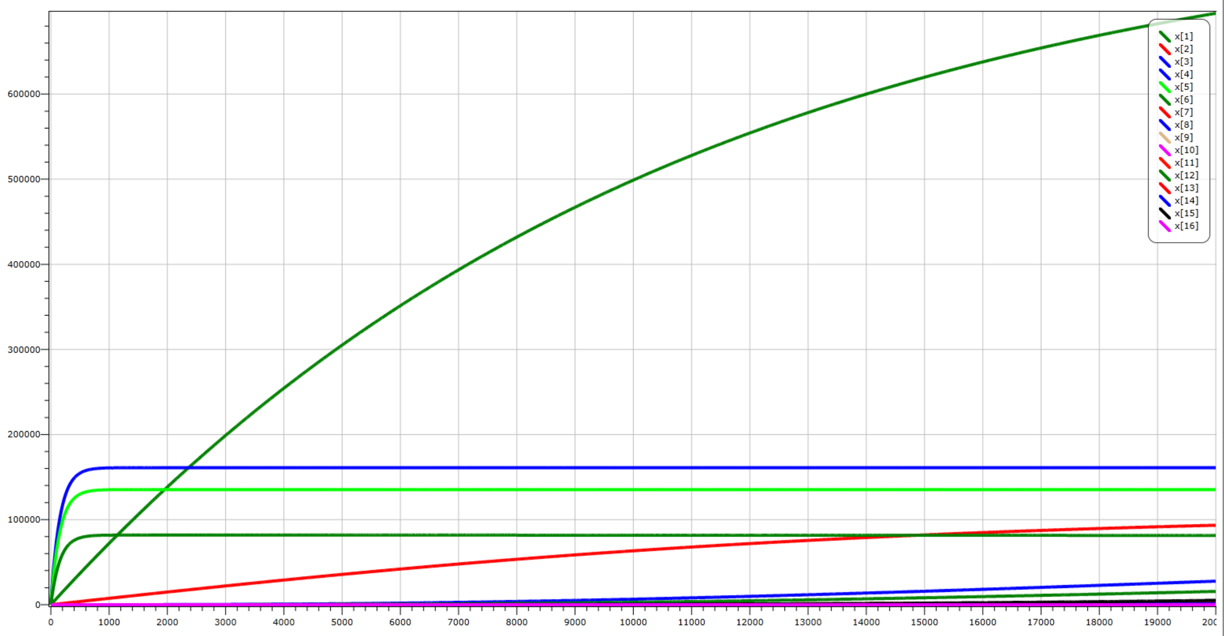
Microsoft Research Solvers is a .NET and Silverlight class library for numerical solution of several types of differential equations. It provides subroutines to integrate vector initial value problem from time t_0 with initial conditions given by vector x_0. Together with DynamicDataDisplay library Solvers allows to construct browser applications that implements different computational models. While Microsoft Research Solvers is in process of publication you may take information about it from [4]. Information about DynamicDataDisplay including link for downloading is available from [3].

Example 1: Oregonator



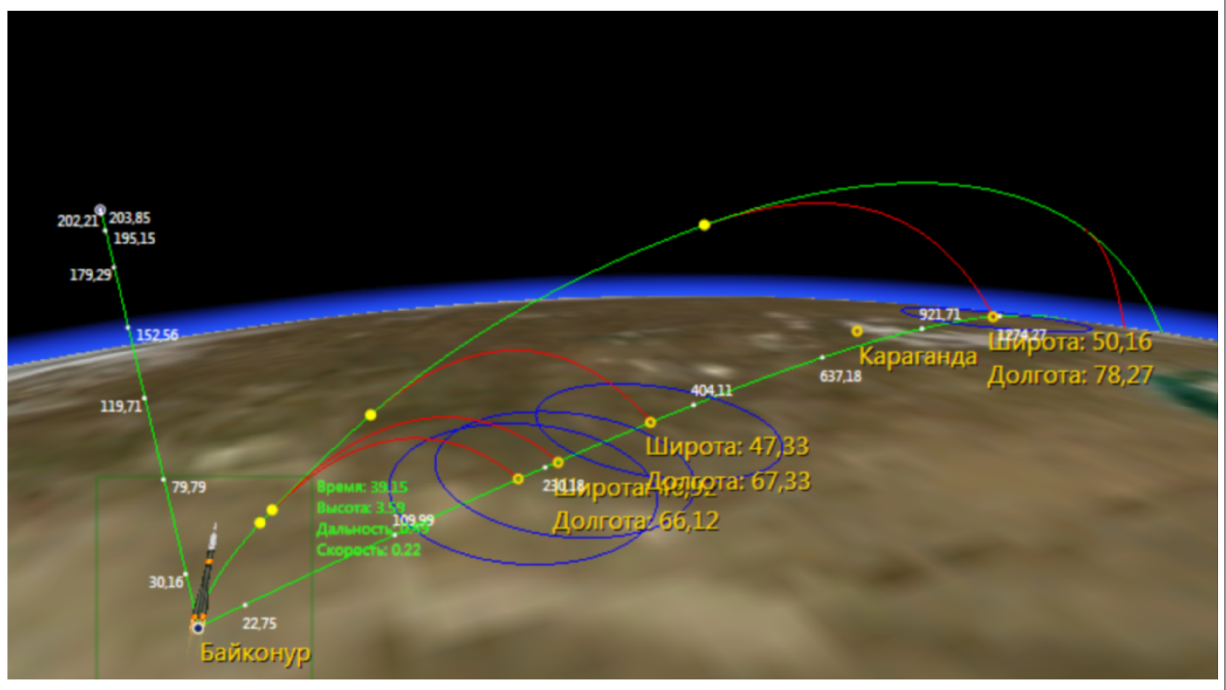
Phase trajectories for a non-linear model of chemical reaction (Belousov-Zhabotinsky (BZ) system).

Example 2: MHC



Phase trajectories for a peptide dynamics model from [2], kindly provided for our experiments by Dr. Dalchau.

Example 3: Mechanical example



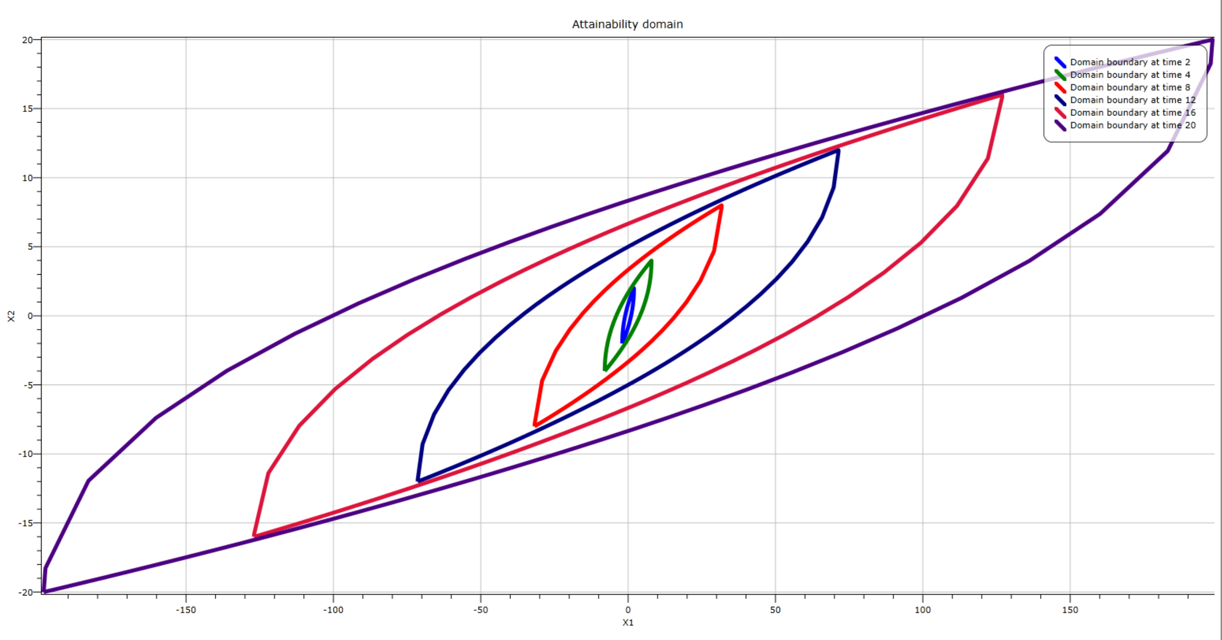
Rocket stages trajectories in Earth's gravity field . This is an example of industrial application of our library.

Using Microsoft Research Solvers library to solve optimal control problems

In this work we use Microsoft Research Solvers as a computational base to implement class library for optimal control problems solving. As a first step we implement a gradient projection based subroutine to solve optimal control problem in Pontryagin form with free right end:

$$\begin{cases} \dot{x} = f(x, t, u), \quad x(t_0) = x_0, \\ x \in R^n, \quad u \in U \subset R^r, \\ J = \displaystyle\int_{t_0}^{T} f_0(t, x, u)dt + \Phi(x(T)) \to \min_{u \in U}. \end{cases}$$

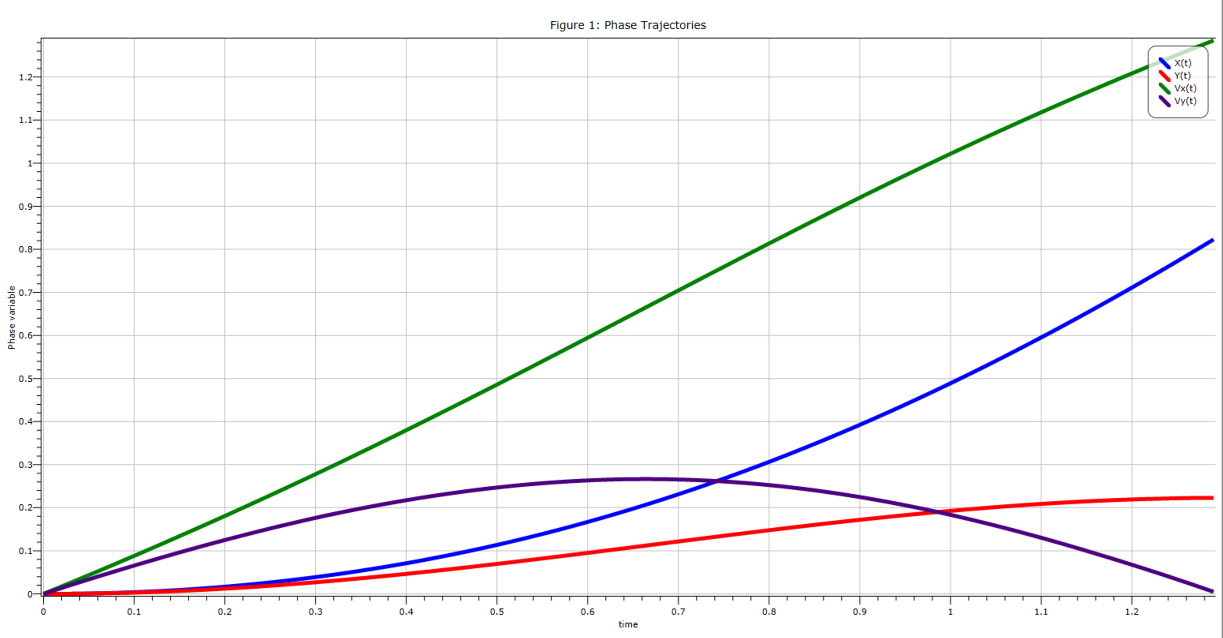
Example 4: "Cart"



Attainability domains for a simple system

$$\begin{cases} \dot{x}_1(t) = x_2(t), \quad x_1(0) = 0, \\ \dot{x}_2(t) = u(t), \quad x_2(0) = 0, \\ u \in U = [-1, 1]. \end{cases}$$

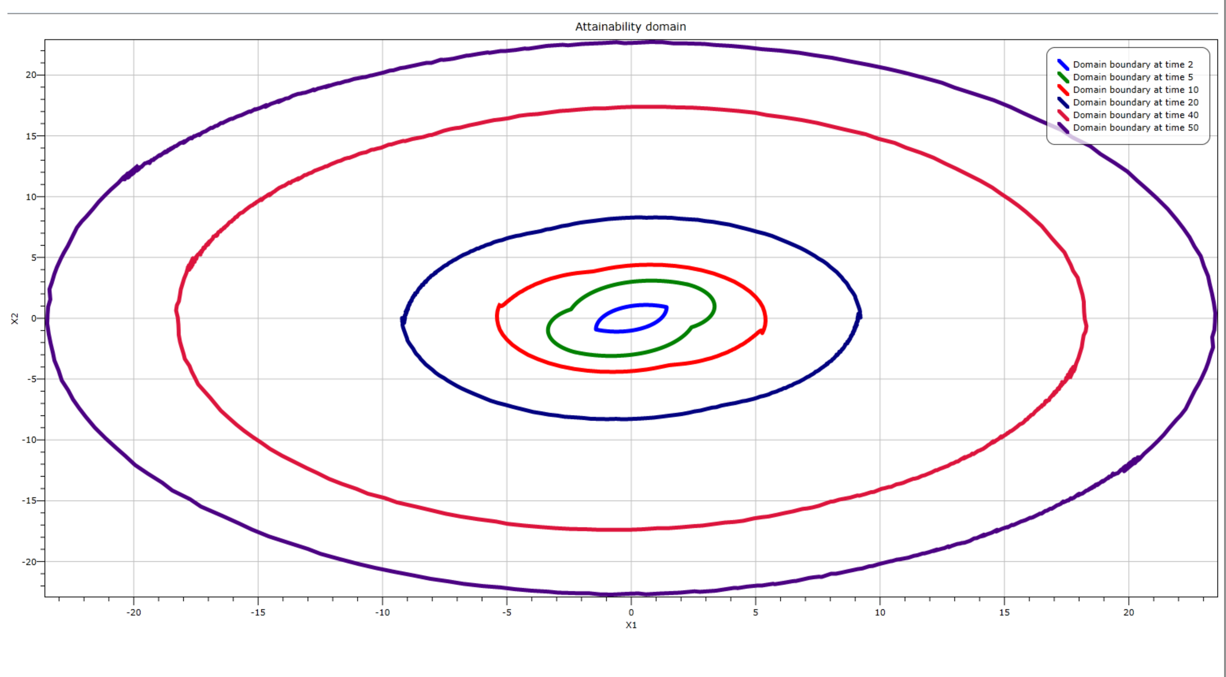
Example 5: "Rocket"



Optimal trajectories for a control system from [1]

$$\begin{cases} \dot{x} = v_x, \quad x(0) = 0, \\ \dot{y} = v_y, \quad y(0) = 0, \\ \dot{v}_x = 1.1 \cos u, \quad v_x(0) = 0, \\ \dot{v}_x = 1.1 \sin u, \quad v_y(0) = 0, \quad u \in [-\pi/2, \pi/2], \\ min_u \left\{ -v_x(1.29) + (y(1.29) - 0.22)^2 + (v_y(1.29)^2) \right\} \end{cases}$$

Example 6: "Pendulum"



Attainability domains for a simple system

$$\begin{cases} \dot{x}_1(t) = x_2(t), \quad x_1(0) = 0, \\ \dot{x}_2(t) = u(t) - x_1(t), \quad x_2(0) = 0, \\ u \in U = [-1, 1]. \end{cases}$$

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- Neil Dalchau, Andrew Phillips, Leonard D. Goldstein , Mark Howarth , Luca Cardelli, Stephen Emmott, Tim Elliott, Joern M. Werner *A Peptide Filtering Relation Quantifies MHC Class I Peptide Optimization*, PLoS Computational Biology, October 2011, Vol. 7, Issue 10, pp. 1–14.
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