1. Narayanaswamy Balakrishnan, McMaster University, Hamilton, Ontario, Canada
   Signatures
   Abstract: In this talk, I will first introduce the notion of "Signatures" and explain its importance in a systematic study of coherent reliability systems. In this regard, I will present some mixture forms and some representation theorems and aging properties. I will then introduce the concept of extended signatures and dynamic signatures and their use in the study of burn-in systems, and their implications in dynamic aging concepts. Finally, I will describe the concept of joint signatures by considering two reliability systems sharing some common components, and present some properties and ordering results of these joint signatures.
2. Vladimir I. Bogachev, Moscow State University, Russia
   Prokhorov's Theorem and the topology of spaces of probability measures
   ABSTRACT: Prokhorov's theorem on necessary and sufficient conditions for weak compactness of families of probability measures, discovered 60 years ago, has become one of the most fundamental achievements of modern probability theory, entered university courses and generated an intensive flow of research related to various generalizations and modifications of this theorem as well as to other problems in the topology of spaces of probability measures. In this talk, we discuss main achievements in this area, open problems with relatively simple formulations, and also connections with certain actively developing directions at the border line of probability theory, measure theory, general topology, and infinite-dimensional analysis. The only prerequisite for understanding the main part of this talk is some acquaintance with basic concepts of measure theory and topology (metric and topological spaces, compacts, continuous functions, Borel measures, Lebesgue's integration) included in standard university programs of the first 4-5 semesters.
3. Igor' S. Borisov, Sobolev Institute of Mathematics, Siberian Branch of RAS, Russia
   Invariance principle and probability inequalities for canonical von Mises' statistics of dependent observations
   ABSTRACT: Hoeffding -type exponential inequalities are obtained for the distribution tails of canonical U- and von Mises' statistics (V-statistics) of arbitrary order based on samples from a stationary sequence of random variables satisfying the ϕ-mixing condition.
   The functional limit theorem, i.e., the invariance principle, is proven for sequences of normalized U- and V-statistics of arbitrary orders with canonical kernels, defined on samples of growing size from a stationary sequence of random variables under the α- or ϕ-mixing conditions. The corresponding limit stochastic processes are described as polynomial forms of a sequence of dependent Wiener processes with a known covariance.
4. Konstantin A. Borovkov, University of Melbourne, Australia
   Continuity Problems in Boundary Crossing Problems
   ABSTRACT: Computing the probability for a given diffusion process to stay under a particular boundary is crucial in many important applications including pricing financial barrier options. It is a rather tedious task that, in the general case, requires the use of some approximation methodology. One possible approach to this problem is to approximate given (general curvilinear) boundaries with some other boundaries, of a form enabling one to relatively easily compute the boundary crossing probability. We discuss results on the accuracy of such approximations for both the Brownian motion process and general time-homogeneous diffusions, their extensions to the multivariate case, and also some contiguous topics.
5. Ekaterina V. Bulinskaya, Moscow State University
   Generalized Renewal Processes. The Limit Behavior and Applications
6. Gerd Christoph, University of Magdeburg, Germany

Differences in Limit Theorems in case of Normal or Non-normal Stable Limit Distributions

ABSTRACT: We consider the asymptotic behavior of standardized sums of independent identically distributed random variables attracted to a stable law. Necessary and sufficient conditions for a certain rate of convergence in the case of the normal limit law are well known, but it is still an open problem for non-normal stable limit laws. First we discuss some reasons for the difficulties in the later case. Then we investigate non-uniform estimates for the remainder terms in approximations with asymptotic expansions, where we again do not have equivalent results for normal and non-normal stable limit laws. Several examples are given to demonstrate the difficulties and the results.

7. Paul Embrechts, ETH Zurich, Switzerland

Risk Aggregation under Dependence Uncertainty

ABSTRACT: Research related to aggregation, robustness, and model uncertainty of regulatory risk measures, for instance, Value-at-Risk (VaR) and Expected Shortfall (ES), is of fundamental importance within quantitative risk management. In risk aggregation, marginal risks and their dependence structure are often modeled separately, leading to uncertainty arising at the level of a joint model. In this paper, we introduce a notion of qualitative robustness for risk measures, concerning the sensitivity of a risk measure to the uncertainty of dependence in risk aggregation. I will give approximations and inequalities for aggregation and diversification of VaR under dependence uncertainty, and derive an asymptotic equivalence for worst-case VaR and ES under general conditions. We obtain that for a portfolio of a large number of risks VaR generally has a larger uncertainty spread compared to ES. The results warn that unjustified diversification arguments for VaR used in risk management need to be taken with much care, and potentially support the use of ES in risk aggregation. This in particular reflects on the discussions in the recent consultative documents by the Basel Committee on Banking Supervision.

8. Yasunori Fujikoshi, Hiroshima University, Japan

Explicit and Computable Error Bounds for Asymptotic Expansions of the Distribution Functions of Some Multivariate Statistics

ABSTRACT: We are interested in two types of asymptotic approximations in multivariate analysis based on n samples of p variables. One is a large-sample asymptotic approximation under the case where n tends to infinity, but p is fixed. The other is a high-dimensional asymptotic approximation under the case where both p and p tend to infinity in a way such that p/n tends to c from [0; 1). Such asymptotic approximations have been obtained, see, e.g., Anderson (2003), Bai and Silverstein (2010), Fujikoshi et al. (2010), Muirhead (1982), Siotani et al. (1985). One side, the results on asymptotic approximations with explicit and computable error bounds are not so many. Some results have been obtained by using error bounds for approximations of the distributions of scale mixtures, multivariate scale mixtures and location and scale mixtures. Its systematic approach is given in Fujikosihi (1993), Fujikoshi et al. (2010). Some other error bounds have been obtained (see, e.g Wakaki (2008)) for high-dimensional approximations of LR statistics for covariance maricies whose moments are expressed in terms of gamma-functions. The purpose of the talk is to review results on explicit and computable error bounds in multivariate statistical analysis mensioned above and the recent results by Wakaki and Fujikoshi (2012) and Yamada et al. (2015).

9. Friedrich Götze, Bielefeld University, Germany

Entropy Expansions in Probability

ABSTRACT: We investigate the convergence of sums of random variables to Gaussian and stable laws in Entropy resp. Fischer-Information distances. In particular we show asymptotic

expansions of such distances in terms of semi-invariants (under minimal assumptions) in the context of classical and free probability.
The results are based on techniques of harmonic and complex analysis for the approximation of classical and free convolutions of densities. The common structure of asymptotic expansions for these and other limit theorems may be explained by a scheme of approximations for sequences of classes of symmetric functions on spaces of increasing dimension.
This is joint work with S. Bobkov, C. Chistyakov and A. Reshetenko.

10. Alexandr S. Holevo, Steklov Mathematical Institute, RAS, Moscow, Russia
    ABSTRACT:
11. Il'dar A. Ibragimov, St. Petersburg Department of V.A. Steklov Institute of Mathematics, RAS, Russia
    ABSTRACT:
12. Peter Jagers, Chalmers University of Technology, Sweden
    Interaction and Limitation in Branching Processes
    ABSTRACT:
13. Bing-Yi Jing, Hong Kong University of Science and Technology, China
    Two-Dimensional Fused-Lasso with non-convex penalty
    ABSTRACT: In this talk, we will investigate feature selections using non-convex penalty. In particular, we study the maximum concavity penalty (MCP) in the context of image reconstruction. We will illustrate that the new method will give much sharper images along the edges and better contrast than alternatives. Furthermore, we will try to develop algorithms for using these non-convex penalties.
14. Gyula O. H. Katona, Afréd Rényi Institute of Mathematics, Hungarian Academy of Sciences, Budapest, Hungary
    Large intersecting families in a finite set
    ABSTRACT: See pdf-file
15. Victor Yu. Korolev, Moscow State University
    Normal Mixture Models: Prokhorov's Ideas and Some Recent Results
    ABSTRACT: See pdf-file
16. Mikhail A. Lifshits, Saint Petersburg State University, Russia
    Least Energy Functions Accompanying Brownian motion
    ABSTRACT: See pdf-file
17. Thomas Mikosh, University of Copenhagen, Denmark
    Asymptotic theory for the largest eigenvalues of the sample autocovariance function of high-dimensional heavy-tailed time series
    ABSTRACT: This is joint work with Richard A. Davis (Copenhagen) and Johann Heiny (Copenhagen). Davis, Mikosch and Pfaffel (2014) studied the limit structure of the sample covariance matrix for linear multivariate time series when the dimension increases with the sample size. We use the same model and assume that the 4th moment of the time series is infinite. This is not uncommon for financial time series, in particular when one studies squares of return series. We study the largest eigenvalues of the sample autocovariance matrices. The main result shows that the point process of the normalized eigenvalues is approximated by a Poisson cluster process, depending on the coefficients of the linear process. Since the results are formulated in terms of converging point processes one may apply continuous mappings to the largest eigenvalues, for example to their sums and ratios. In particular, we apply the limit theory in the setting of a paper by Lam and Yao (2012, AoS) who suggested a tool for determining the number of relevant eigenvalues in the context of high-dimensional financial time series analysis.

18. Ernst L'vovich Presman, Central Economics and Mathematics Institute, RAS, Moscow
    Prokhorov Memorial Lecture
19. Alexander I. Sakhanenko, Novosibirsk State University, Russia
    ON ASYMPTOTICS AND ESTIMATES FOR GENERALIZED PROKHOROV DISTANCE IN INVARIANCE PRINCIPLE
    ABSTRACT: First, in the talk we will revisit the Invariance Principle and Prokhorov distance. A survey of convenient estimates in the Invariance Principle will be presented. Then we will compare the existing methods to obtain such estimates.
    We will pay special attention to the exact asymptotics of Prokhorov distance and its generalizations. Several new results in this direction will be presented. We will conclude with a novel idea on how to obtain the exact asymptotics.
    It should be stressed that the first upper estimate of this type and the first coupling method were introduced in the fundamental paper of Yu. V. Prokhorov (Theory Prob. and Appl. 1956. pp.157-214) together with Prokhorov distance and many other ideas.
20. Qi-Man Shao, The Chinese University of Hong Kong, China,
    Cramer Type Moderate Deviations of Normal and Non-normal Approximation
    ABSTRACT: The Cramer type moderate deviation theorem quantities the accuracy of the relative error in the distribution approximation. It has been used to measure the accuracy of an estimated p-value and the theorem has played an important role in the multi-scale hypothesis tests, especially in the study of controlling the false discovery rate. In this talk we shall review recent developments on the Cramer moderate deviation results for normal and non-normal approximation. Applications will also be discussed.
21. Irina G. Shevtsova, Moscow State University
    Some Recent Results in the Field of Accuracy of the Normal Approximation to Sums of Independent Random Variables
22. Albert N. Shiryaev, Steklov Mathematical Institute, RAS, Moscow, Russia
23. Alexander N. Tikhomirov, Komi Scientific Center of Ural Branch of RAS, Russia
    Inequalities for the Moments of Quadratic Forms
    ABSTRACT: We consider Khinchuin type inequalities for quadratic forms in independnet ranom variables under moment conditions.
    We obtain the inequalities of moment of quadratic forms for truncated random variables as well. The applications of such inequalities for random matrices are considered.
24. Vladimir V.Ulyanov, Alexey A.Naumov, Lomonosov Moscow State University
    On Polynomials in Random Elements
    ABSTRACT: We are interested in the bounds for characteristic functions of polynomials in random vectors. One of the common approach is to use symmetrization inequalities. However, the bounds can be made much sharper by applying technique and inequalities for trigonometric sums and integrals known in number theory provided that some conditions on the type of the distribution of the random vector are met. Stochastic generalization of the Vinogradov mean value theorem is also presented. We use the results obtained by Yu.V.Prokhorov with coauthors. The second part of the talk is devoted to general asymptotic expansions for a class of sequences of symmetric functions of many variables. Applications to classical and free probability theory are discussed.
25. Andrei Yu. Zaitsev, St. Petersburg Department of V.A. Steklov Institute of Mathematics, RAS, Russia
    Some results concerning the sums of independent random vectors
    ABSTRACT: See pdf-file

Large intersecting families in a finite set

Gyula O.H. Katona
Rényi Institute, Budapest, Hungary

Abstract

We will consider families $\mathcal{F}$ of subsets of $[n] = \{1, 2, \ldots, n\}$. Suppose that the intersection $F \cap G$ is non-empty for any two members $F, G \in \mathcal{F}$. Such a family is called *intersecting*. It is easy to see that $|\mathcal{F}| \leq 2^{n-1}$ holds for an intersecting family and this is the best possible bound. The modified problem when the members of $\mathcal{F}$ have exactly k elements (k-uniform family) is less trivial. It was solved in 1961 by Erdős, Ko and Rado. The largest intersecting family in this case has $\binom{n-1}{k-1}$ members when $2k \leq n$ (otherwise the problem is trivial).

We will show some older and and some recent results of this type. E.g. the following variant. Call the family $\mathcal{F}$ *neighboring* if any two members F, G are either intersecting or they have neighboring elements $f \in F, g \in G$ such that $|f - g| = 1$. We have asymptotic results on the maximum size of a neighboring family. Probabilistic methods are used in the proof of this result jointly achieved by David Retek.

Normal mixture models: Prokhorov's ideas and some recent results

Victor Korolev

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Institute for Informatics Problems, Russian Academy of Sciences

Abstract

The wide range of mathematical interests of Yu. V. Prokhorov involved normal mixture models. He considered these models both as a tool to prove new results not linked with them directly and as a direct object of his study. Some results dealing with normal mixture models obtained by his PhD students are given.

Also some recent results are presented. Special attention is paid to mixture representations of the Weibull distribution. Product representations are obtained for random variables with the Weibull distribution in terms of random variables with normal, exponential and stable distributions yielding scale mixture representations for the corresponding distributions. Main attention is paid to the case where the shape parameter γ of the Weibull distribution belongs to the interval $(0, 1]$. The case of small values of γ is of special interest, since the Weibull distributions with such parameters occupy an intermediate position between distributions with exponentially decreasing tails (e. g., exponential and gamma distributions) and heavy-tailed distributions with Zipf–Pareto power-type decrease of tails. It is demonstrated that if $\gamma \in (0, 1]$, then the Weibull distribution is a mixed half-normal law, and hence, it can be limiting for maximal random sums of independent random variables with finite variances.

It is demonstrated that product representations for Weibull-distributed random variables considerably simplify the proofs of some results dealing with the infinite divisibility of the Weibull distribution and the explicit representations of the moments of stable laws.

It is also demonstrated that the symmetric two-sided Weibull distribution with $\gamma \in (0, 1]$ is a scale mixture of normal laws. Necessary and sufficient conditions are proved for the convergence of the distributions of extremal random sums of independent random variables with finite variances and of the distributions of the absolute values of these random sums to the Weibull distribution as well as of those of random sums themselves to the symmetric two-sided Weibull distribution. These results can serve as theoretical grounds for the application of the Weibull distribution as an asymptotic approximation for statistical regularities observed in the scheme of stopped random walks used, say, to describe the evolution of stock prices and financial indexes. Also, necessary and sufficient conditions are proved for the convergence of the distributions of more general regular statistics constructed from samples with random sizes to the symmetric two-sided Weibull distribution.

Least Energy Functions Accompanying Brownian motion

Mikhail Lifshits
St.Petersburg University and Linköping University

In the language of Control Theory the problem can be partially described as "You lead on a leash a dog performing Brownian motion. How to do it in a sustainable way (by expending minimal energy) ?"

We investigate how well can one approximate Brownian motion by smooth functions on long intervals of time. Consider uniform norms

$$||h||_T := \sup_{0\le t\le T} |h(t)|, \qquad h \in \mathbb{C}[0,T],$$

and Sobolev-type energy norms

$$|h|_T^2 := \int_0^T h'(t)^2 dt, \qquad h \in AC[0,T].$$

Let W be a Brownian motion. We are mostly interested in its approximation characteristic

$$I_W^0(T,r) := \inf\big\{|h|_T; h \in AC[0,T], ||h-W||_T \le r, h(0)=0, h(T)=W(T)\big\}.$$

The unique function at which the infimum is attained is called *taut string* with fixed end. This notion appears in many branches of pure and applied math. Our typical result is as follows.

Theorem 1 *There exists a constant* $\mathcal{C} > 0$ *such that for any fixed* $r > 0$ *we have*

$$\frac{r}{T^{1/2}}\ I_W^0(T,r) \xrightarrow{a.s.} \mathcal{C}, \qquad \text{as } T \to \infty.$$

We give analytical and numerical bounds for $\mathcal{C}$ and solve an adapted version of our problem which belongs to optimal control theory. Furthermore, we evaluate average least energy for fixed T and replace the rigid restriction $||h-W||_T \le r$ by introducing a penalty function.

This is a joint work with Z. Kabluchko and E. Setterqvist.

SOME RESULTS CONCERNING THE SUMS OF INDEPENDENT RANDOM VECTORS

ANDREI YU. ZAITSEV
ST. PETERSBURG DEPARTMENT OF STEKLOV MATHEMATICAL INSTITUTE,

We discuss some recent results [1]–[4] concerning the sums of independent random vectors.

The problem of the approximation of convolutions by accompanying laws in the scheme of series satisfying the infinitesimality condition is considered [4]. It is shown that the quality of approximation depends essentially on the choice of centering constants.

In the paper [2], it is shown that the results obtained earlier for the rate of approximation of convolutions of probability distributions by the accompanying infinitely divisible compound Poisson laws may be interpreted as estimates of the rate of approximation of a sample by a Poisson point process. The most interesting results are obtained for a scheme of rare events.

Concentration functions of n-fold convolutions of probability distributions is shown [3] to exhibit the following behavior. Let $\varphi(n)$ be an arbitrary sequence tending to infinity as n tends to infinity, and $\psi(x)$ be an arbitrary function tending to infinity as x tends to infinity. Then there exists a probability distribution F of a random variable X such that the mathematical expectation $\mathbf{E}\,\psi(|X|)$ is infinite and, moreover, the upper limit of the sequence $n^{1/2}\varphi(n)Q(n)$ is equal to infinity, where $Q(n)$ is the maximal atom of the n-fold convolution of distribution F. Thus, no infinity conditions imposed on the moments can force the concentration functions of n-fold convolutions decay essentially faster than $o(n^{-1/2})$.

The paper [1] deals with studying a connection of the Littlewood–Offord problem with estimating the concentration functions of some symmetric infinitely divisible distributions.

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Explicit and Computable Error Bounds for Asymptotic Expansions of the Distribution Functions of Some Multivariate Statistics

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Summary

Explicit and Computable Error Bounds

For the distributions of most multivariate statistics, we use their asymptotic approximations, since their exact distributions are complicated or can not be obtained. So, it is important to derive asymptotic distributions and examine their accuracies. Ideally it is hoped to derive explicit and computable error bounds for asymptotic approximations in the following sense. Let $F_n(x)$ be the distribution function depending on some parameter, typically a sample size. Suppose that $F_n(x)$ can be approximated by an asymptotic expansion with k terms, in a common form

$$G_{k,n}(x) = G(x) + \sum_{j=1}^{k-1} n^{-j/2} p_j(x) g(x),$$

where $G(x)$ is the limiting distribution function of $F_n(x)$ as $n \to \infty$, $g(x)$ is the pdf of $G(x)$, and $p_j(x)$'s are suitable polynomials. Usually, it is shown that

$$R_{n,k}(x) \equiv F_n(x) - G_{k,n}(x) = \mathrm{O}(n^{-k/2}),$$

which is equivalent to saying that there exist a positive constant c_k and a positive number N_k such that

$$|R_{n,k}(x)| \leq n^{-k/2} c_k, \quad \text{for } n \geq N_k.$$

In some cases, F_n depends on the population parameter $\theta \in \Theta$. If this is so, c_k and N_k may depend on θ, i.e., $c_k = c_k(\theta)$, $N_k = N_k(\theta)$. However, usually c_k and N_k are unknown. It is hoped to find an explicit constant and computable c_k or an explicit and computable function of θ such that for all n (except several values of n),

$$|R_{n,k}(x)| \leq n^{-k/2} c_k.$$

We call such an explicit and computable error bound simply error bound.

Purposes of the Present Paper

We are interested in two types of asymptotic approximations in multivariate analysis based on n samples of p variables. One is a large-sample asymptotic approximation under the case where n tends to infinity, but p is fixed. The other is a high-dimensional asymptotic approximation under the case where both p and p tend to infinity in a way such that $p/n \to c \in [0, 1)$. Such asymptotic approximations have been obtained, see, e.g., Anderson (2003), Bai and Silverstein (2010), Fujikoshi et al. (2010), Muirhead (1982), Siotani et al. (1985). One side, the results on asymptotic approximations with explicit and computable error bounds are not so many. Some results have been obtained by using error bounds for approximations of the distributions of scale mixtures, multivariate scale mixtures and location and scale mixtures. Its systematic approach is given in Fujikosihi (1993), Fujikoshi et al. (2010). Some other error bounds have been obtained (see, e.g Wakaki (2008)) for high-dimensional approximations of LR statistics for covariance maricies whose moments are expressed in terms of gamma-functions.

The purpose of this paper is to review results on explicit and computable error bounds in multivariate statistical analysis mensioned above and the recent results by Wakaki and Fujikoshi (2012) and Yamada et al. (2015). .

Main Results

(1) Estimators in Growth Curve Model, etc. The distributions of estimators of mean parameters in growth curve model and profile analysis can be expressed a scale mixture, i.e. $X = SZ$, where $S > 0$, S and Z is independent. In most case Z is normal, or chi-square variable. In fact, we develop a theory of approximating the distribution of

$$X = S^{\delta} Z,$$

where $\delta = \pm 1$ is a parameter used to distinguish two types of mixtures: $X = SZ$ and $(1/S)Z$. For the results, see, Fujikoshi (1985, 1988), Shimizu and Fujikoshi (1997), Fujikoshi et al. (2010).

(2) MANOVA Tests. The null distributions of Lawley-Hoteling's T_0^2 test and Likelihood ratio test in MANOVA expressed as

$$T_{LH} = n\mathrm{tr} S_h S_e^{-1}, \quad T_{LR} = -n \log\{|S_e|/|S_e + S_h|\},$$

where S_e and S_h are independently distributed as a Wishart distribution $W_p(n, \Sigma)$ and a Wishart distribution $W_p(q, \Sigma)$, respectively. Under a large-sample asymptotic framework, the limiting distributions of T_{LH} and T_{LR} are a chi-square distribution with pq-degrees of freedom when n is large. Error bounds are obtained by expressing these statistics in terms of a multivariate scale mixture, and using error bounds evaluated in L_1-norm. For the results, see, Fujikoshi et al. (2005), Fujikoshi and Ulyanov (2006), Fujikoshi et al. (2010).

(3) Statistics with a Class of Moments. The null distribution of $\Lambda = |S_e|/|S_e + S_h|$ is expressed as a Lambda distribution denoted as $\Lambda_p(q, n)$. The hth moment of $W = \Lambda^{n/2}$ is

$$\mathrm{E}[W^h] = \mathrm{E}[\Lambda^{nh/2}] = K \frac{\prod_{j=1}^{p} \Gamma[\frac{1}{2}n(1+h) + \frac{1}{2}(1-j)]}{\prod_{j=1}^{p} \Gamma[\frac{1}{2}n(1+h) + \frac{1}{2}(q+1-j)]},$$

where K is a constant such that $\mathrm{E}(W^0) = 1$. The moment is a special case of a statistic $W (0 \le W \le 1)$ with

$$\mathrm{E}(W^h) = K \left(\frac{\prod_{k=1}^{b} y_k^{y_k}}{\prod_{j=1}^{a} x_j^{x_j}} \right)^h \frac{\prod_{j=1}^{a} \Gamma[x_j(1+h) + \xi_j]}{\prod_{k=1}^{b} \Gamma[y_k(1+h) + \eta_k]}, \quad h = 0, 1, \ldots,$$

where $\sum_{j=1}^{a} x_j = \sum_{k=1}^{b} y_k$, with the correspondence $a = p,\ x_q = \frac{1}{2}n,\ \xi_q = \frac{1}{2}(1-q),\ c_q = \frac{1}{2},\ b = p,\ y_k = \frac{1}{2}n,\ \eta_k = \frac{1}{2}(q+1-k),\ d_k = \frac{1}{2}$. It is well known that some of LR statistics for mean parameters and covariance matrices have such moments. Then, we have high-dimensional asymptotic approximations as well as large-sample asymptotic approximations. For high-dimensional asymptotic approximations, see Fujikoshi et al. (2010). Recently, error bounds have been obtained for some high-dimensional asymptotic approximations. For the results, see Wakaki (2008), Kato et al. (2010), Akita et al. (2010).

(4) LR test for Additional Information in Cananonical Correlation Anaalysis. In canonical correlation analysis of p-variate random vector $\boldsymbol{x}$ and q-variate random vector $\boldsymbol{y}$, Fujikoshi (1982) obtained the likelihood ratio criterion λ for testing an additional information hypothesis on $p_2(= p - p_1)$-subvector of $\boldsymbol{x}$ and $q_2(= q - q_1)$-subvector of $\boldsymbol{y}$, based on a random sample of size $N = n + 1$. Then it is known that λ is expressed as

$$L = -(2/N)\log \lambda = L_{(1)}L_{(2)},$$

where the null distributions of $L_{(1)}$ and $L_{(2)}$ are independently distributed as $\Lambda_{p_2}(q, n - p_1 - q)$ and $\Lambda_{q_2}(p_1, n - p_1 - q_1)$, respectively. The high–dimensional asymptotic distribution of λ has been studied in Sakurai (2010), Wakaki and Fujikoshi (2012) when the sample size and the dimensions are large. For error bounds for high-dimensional approximations, see Wakaki and Fujikoshi (2012).

(5) Linear and Quadratic Discriminant Functions. It is known that the linear discriminant function W can be expressed a location and scale mixture, i.e., $W = VZ + U$, where $Z \sim N(0, 1)$, $V > 0$, and Z and (U, V) are independent. Then, the result was applied to obtain error bounds for high-dimensional asymptotic approximations as well as large-sample approximations for the distribution of W. For these results, see Fujikoshi (1993, 2000), Fujikoshi and Ulyanov (2006), Fujikoshi et al. (2010). Recently Yamada et al. (2015) pointed that the quadratic discriminant function can be also expressed as a a location and scale mixture, and gave error bounds for its high-dimensional approximations.

Error Bounds to be Obtained in MANOVA

The three major test statistics in MANOVA are LR (likelihood ratio) test; T_{LR}, Lawley–Hotelling's statistic; T_{LH}, Bartlett–Nanda–Pillai's statistic; $T_{BNP} = n\mathrm{tr}S_h(S_e + S_h)^{-1}$, where S_e and S_h are independently distributed as a Wishart distribution $W_p(n, \Sigma)$ and a Wishart distribution $W_p(q, \Sigma)$, respectively. For high-dimensional approximations, see Wakaki et al. (2014). Error bounds have been obtained for high-dimensional and large-sample approximations of T_{LR} and large-sample approximations of T_{LH}. However, error bounds are not obtained for high-dimensional approximations of T_{LR}, and high-dimensional and large-sample approximations of T_{BNP}. It is hoped that these unknown problems are solved.

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