TITLES AND ABSTRACTS

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POISSONIZATION INEQUALITIES FOR SUMS OF INDEPENDENT B-VALUED RANDOM VARIABLES

Let $X_1, X_2, \dots$ be i.i.d. random variables taking values in a separable Banach space $(\mathcal{B}, \|\cdot\|)$. Denote by $Pois(\mu)$ the compound Poisson distribution with Lévy measure μ:

$$Pois(\mu) := e^{-\mu(\mathcal{B})} \sum_{k=0}^{\infty} \frac{\mu^{*k}}{k!},$$

where μ^{*k} is the k-fold convolution of a finite measure μ with itself; μ^{*0} is the unit mass concentrated at zero. Denote by $\tau(\mu)$ a r.v. with the distribution $Pois(\mu)$.

Put $S_n := \Sigma_{i\le n} X_i$, $n \ge 0$, with $S_0 = 0$. The compound Poisson distribution with Lévy measure $\mu \equiv \mu_n := n\mathcal{L}(X_1)$ is called the *accompanying infinitely divisible law* for the distribution of S_n. In other words,

$$Pois(\mu_n) = \mathcal{L}(S_{\pi(n)}),$$

where $\pi(n)$ is a Poisson random variable with mean n, which is independent of $\{X_i\}$.

In the talk, we discuss moment inequalities of the form

$$\mathbf{E}F(S_n) \le C_o \mathbf{E}F(\tau(\mu_n)), \tag{1}$$

where F is a measurable functional on $(\mathcal{B}, \|\cdot\|)$ and $C_o \ge 1$ is a constant not depending on n. In particular, such inequalities for empirical processes will be considered.

For the first time, moment inequalities of the form (1) were found by Yu. V. Prohorov in 1962. He proved (1) with $C_o = 1$ for all even-power functions $F(x) = x^{2m}$ and real-valued symmetrically distributed random variables $\{X_i\}$.

Ekaterina V. Bulinskaya, Moscow State University, Russia

New Applied Probability Models and Optimization Problems

The aim of the talk is investigation of the new applied probability models which appeared during the last ten years. It is well known that the models arising in such applications as insurance, finance, queuing, inventory and dams theory, population dynamics, communication networks, reliability and many others have input-output character. Hence, they are described by the planning horizon $T \le \infty$, input, output and control processes, as well as a functional specifying the system structure and functioning mode. In order to evaluate the performance quality of the system one has to introduce an objective function (risk measure). According to the choice of risk measure it is possible to ascertain two main approaches, namely, reliability and cost ones. In the first case, the researcher is interested in the maximization of the system uninterrupted performance or minimization of ruin probability. In the second case, the goal is minimization of (expected) loss or maximization of (expected) profit. It is possible to introduce more intricate risk measures.

Along with establishing the optimal and asymptotically optimal control for several continuous- and discrete-time models we study the systems asymptotical behavior and their stability. Simulation problems are tackled as well.

Gerd Christoph, University of Magdeburg, Germany

Asymptotic Expansions for Multivariate Statistics Based on Random Size Samples

In practice, we often encounter situations where a sample size is not defined in advance and can be a random value. The randomness of the sample size crucially changes the asymptotic properties of the underlying statistic. But also a random or non-random scaling factor at the statistic, which is based on a sample with random sample size, it influences the limit distribution. For random mean and random median second order Chebyshev-Edgeworth-type expansions are considered. To estimate a location parameter of the underlying sample with random sample size one could use the random mean but for its second order expansion more than the fourth moment of the independent identically distributed observations X_1, X_2, … is required. For heavy tailed distributions of X_1 with tail index not larger than 4 such second order Edgeworth expansions of the random mean do not exist. For the random median some regularity assumptions on the density p(x) of X_1 are required. In [1] the second order asymptotic expansion for the median from a sample with non-random sample size is proved. Together with the second order expansions for some random sizes in [2] and the transfer theorem in [3] we obtain second order expansions for the median of a samples with random sizes.

References

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Manuel L. Esquível, NOVA University Lisbon,Portugal

An example of a Financial Market Model obtained by Euler Discretization of a Continuous Model

We focus our interest on the study of discrete time financial models with one risky asset and a risk free asset that may thought to result as a discretization of suitable continuous time models. We compare the pricing results obtained with these models, with results obtained from the related continuous time models. Our approach relies on some known important results describing a particular class of discrete time models -- the conditionally Gaussian models introduced by A. N. Shiryaev -- a class that contains many interesting instances. We aim at a better understanding of the implications of the discretization procedures which are inevitable, both at the parameter estimation and derivative price computation moments, by reason of the computational implementations.

Yasunori Fujikoshi, Hiroshima University, Japan

Asymptotic Results on Joint Variable and Rank Selection Methods in High-dimensional Multivariate Regression Model

This talk is based on a joint work with Dr. T. Sakurai which was presented in the 2019 Japanese joint statistical meeting. This paper is concerned with the problem of selecting the explanatory variables and a rank simultaneously in a high-dimensional multivariate regression model with p response variables and k explanatory variables, based on a sample of size n. Such joint selection problem has been studied in two cases (i) $\boldsymbol{\Sigma} = \sigma^2 \mathbf{I}_p$, and (ii) $\boldsymbol{\Sigma} > \mathbf{O}$. For the case (i), some methods have been proposed by Bunea et al. (2012), Chen et al. (2012) and Fujikoshi (2016). The method by Fujikoshi (2016) was extended for the case (ii). In this paper we use the following usual two-step procedures: (1) First select variables, then rank. (2) First select rank and then variables, (3) Select variables and rank independently. We propose the following methods in each steps of these procedures. For a selection of explanatory variables, we apply KOO(Kick-One-Out) method, and for a selection of rank, we use a general information criterion. Consistency properties are given for the selection methods in each steps as well as the combined procedures under high-dimensional asymptotic frameworks (A1) : $p/n \to c_1$ and (A2) : $p/n \to c_1,\ k/n \to c_2$. We also give some asymptotic results on the test statistics and the characteristic roots related to these problems, under high-dimensional frameworks.

Friedrich Götze, Bielefeld University, Germany

Concentration of Measure and Entropic Convergence

We study 'higher' order concentration of measure bounds for functionals on the sphere, Euclidean and discrete spaces. These general results will be applied to the distribution of weighted sums with dependencies and to distribution questions for spin systems and unbounded functionals of polynomial type. Furthermore, we discuss the entropic convergence to the Poisson law measured in relative entropy based divergences. This includes the full hierarchy of Renyi type divergences.

Vladimir I. Lotov, Novosibirsk State University and Sobolev Institute of Mathematics of RAS, Russia

Random walks with two levels of control

We study the properties of an oscillating random walk with two regulatory barriers, upon reaching them the distribution of jumps changes. The Laplace-Stieltjes transforms of the stationary and pre-stationary distributions of the walk are found in terms of the Wiener-Hopf factorization. The possibilities of the transforms inversion are discussed. Under Cramer-type condition, asymptotic representations of these transforms are found for widening regulatory band.

The regulation is designed to keep the range of trajectories under control. At the same time, the structure of the process allows the trajectories to stay out of the band for some time. Limit theorems for the distribution of the maximal possible excess over upper barrier are established as well.

Similar results are also obtained for a stochastic process with switching between two stationary processes with independent increments.

Yakov Yu. Nikitin, Saint-Petersburg University and Higher School of Economics, Russia

Goodness-of-fit and symmetry tests based on characterizations, and their efficiency

A survey of goodness-of-fit and symmetry tests based on the characterization properties of distributions is presented. This approach became popular in recent years. In most cases the test statistics are functionals of U-empirical processes. The limiting distributions and large deviations of new statistics under the null hypothesis are described. Their local Bahadur efficiency for various parametric alternatives is calculated and compared with each other as well as with various previously known tests. We also describe new directions of possible research in this domain.

Shige Peng, Shandong University, China

Law of large numbers and central limit theorem in cases of uncertainty of probabilities

Limit theorem is a very active research domain in probability theory and statistics, in which the law of large numbers and central limit theorem (LLN & CLT) played a central role. In this talk, we present some recent rapid developments of LLN and CLT in situations where the probability measure itself has non negligible uncertainty. We also discuss its application in dynamical data analysis.

Peter Major, Alfréd Rényi Institute of Mathematics, Hungary

Non-central limit theorems for non-linear functionals of vector valued Gaussian stationary random sequences

Abstract. Our main problem is the description of possible limit theorems for non-linear functionals of vector valued Gaussian stationary random sequences. More explicitly, we are interested in the following problem.

Let $X(n) = (X_1(n), \dots, X_d(n))$, $n = \dots, -1, 0, 1, \dots$, be a stationary sequence of d-dimensional Gaussian random vectors, and let us have a function $H(x_1, \dots, x_d)$ of d variables. Define with their help the random variables

$$Y_n = H(X_1(n), \dots, X_d(n)), \quad n = \dots, -1, 0, 1, \dots$$

and their normalized partial sums

$$S_N = \frac{1}{A_N} \sum_{n=0}^{N-1} Y_n, \quad N = 1, 2, \dots$$

with an appropriate norming constant A_N. Prove a new type of limit theorem for S_N (with a non-Gaussian limit) under appropriate conditions on the stationary sequence of d-dimensional random vectors and function $H(\cdot)$.

In the scalar valued case $d = 1$ we have proved with R. L. Dobrushin such a result in [2]. Now we want to prove its multivariate version. A. M. Arcones claimed to do this in his paper [1], but there are some serious problems with his proof.

Our proof with Dobrushin in [1] was based on Dobrushin's theory about the Wiener–Itô integral representation of non-linear functionals of a stationary Gaussian random sequence. But this theory worked only for scalar valued random sequences. A. M. Arcones disregarded this fact, and he worked freely with some multivariate analogues of Dobrushin's results which had no proof. In particular, he expressed the limit in his limit theorem with the help of such random integrals with respect to a multivariate random spectral measure which have not been defined.

Our goal is to build up a correct theory about the Wiener–Itô integral representation of non-linear functionals of multivariate stationary Gaussian random sequences and to show how one can prove the desired results with their help. (See [3].) In particular, we are interested in the question when we get a non-Gaussian and when a Gaussian limit in our limit theorem.

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Ernst L. Presman, CEMI RAN, Russia

On modifications of Lindeberg and Rotar conditions in Central Limit Thorem.

We discuss the history of the Central Limit Theorem in a classical and nonclassical setting and give in a sense a simple proof of Lindeberg-Feller theorem. Lindeberg characteristic can be written in the following form: $L_n(\varepsilon) = \sum \mathrm{E}(X_{nk}^2 I(|X_{nk}| > \varepsilon)$, where $X_{nk}, 1 \le k \le k_n$, are the normalized terms of the sum. For the case of nonclassical setting Rotar introduced an analogue of Lindeberg characteristic where instead of the second ε-tail moment he uses the second ε-tail absolute difference pseudo-moment. We present the following modification of the Lindeberg characteristic: $L_n^b = \sum \mathrm{E}(X_{nk}^2 b(X_{nk}))$, where $b = b(x) \in B$, $B-$ is very broad class of functions (in particular $\max[x^\alpha, 1] \in B$ for any α>0). We prove that the following three conditions are equivalent: a) $L_n^{b_0} \to 0$ for some $b_0 \in B$, b) $L_n(\varepsilon) \to 0$ for any $\varepsilon > 0$, c) $L_n^b \to 0$ for any $b \in B$.We introduce also a similar modification of Rotar characteristic and proof a similar statement for nonclassical setting.

Alexander V. Prokhorov, Faculty of Mechanics and Mathematics of MSU

Ю.В. Прохоров в жизни

Albert N. Shiryaev, Steklov Mathematical Institute of RAS, Russia

Prokhorov's theorem

Vladimir V. Ulyanov, Faculty of Computational Mathematics and Cybernetics of MSU

Gaussian measures of large balls in statistical inference

First, we consider some problems on Gaussian measures studied by Yu.V.Prokhorov and that are still open. Then we derive tight non-asymptotic bounds for the Kolmogorov distance between the probabilities of two Gaussian elements to hit a ball in a Hilbert space. The key property of these bounds is that they are dimension-free and depend on the nuclear (Schatten-one) norm of the difference between the covariance operators of the elements and on the norm of the mean shift. The obtained bounds significantly improve the bound based on Pinsker's inequality via the Kullback–Leibler divergence. We also establish an anti-concentration bound for a squared norm of a non-centered Gaussian element in a Hilbert space.

We present a number of examples motivating our results and applications of the obtained bounds to statistical inference and to high-dimensional CLT

Dmitriy N. Zaporozhets, St. Petersburg Department of Steklov Institute of Mathematics of RAS, Russia

Approximation of sums of random vectors by infinitely divisible distributions

We consider a multidimensional generalization of the Kolmogorov theorem on the approximation of sums of independent arbitrary distributed random vectors by infinitely divisible distributions. The talk is based on the joint work with Friedrich Goetze and Andrei Zaitsev.